Embedding Deontic Logic Operators from a Large Ontology in Higher-order Logic*

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Abstract

Deontic Logic is the study of modal logic which takes formulas and applies either Obligation (O), Permission (P), or Forbiddance/Prohibition (F) over them. It has been discussed at length by many authors, but with the advent of automated theorem provers we are able to test and implement large scale deontic theories. Much prior work in this area has considered deontics as small examples, largely separate from use in the context of large scale knowledge representation. This paper looks at the results of testing deontic axioms and systems with example problems using Kripke semantics and possible worlds over accessibility relations in the context of a large ontology. Examples have been written using the Suggested Upper Merged Ontology (SUMO), then automatically translated to TPTP Typed Higher-order Form (THF) as an embedding of a quantified multi-modal logic in classical higher order logic. The theories were then tested with LEO-III-STC for type conflicts and run in Vampire.

Systems ranging from K to S5 were tested on example problems written in THF with their translated SUMO and original plain English in comments. Example problems were for each deontic attribute (O, P, and F), testing *conditionality* (obligations that require a condition versus those that do not), and evaluating existential versus universal conjectures. While *conditionality* was relatively straightforward to implement and test, a system that aligned with what is conceptually correct. On the other hand, querying whether an obligation holds for all worlds versus just some worlds (universal versus existential) was more difficult. Using existential quantification over worlds in the query was the approach chosen for modal operators O and F. We also consider the notion of a sanction in a Contrary To Duty (CTD) scenario, which is one in which obligations or prohibitions are not obeyed.

Keywords

Deontic Logic, Kripke semantics, Suggested Upper Merged Ontology, TPTP,

1. Introduction

We are implementing a quantified multi-modal logic in the context of a large formal ontology. As our first step, we have implemented an embedding of deontic logic in classical higher-order logic with simple type theory. We use the SUMO ontology [1, 2], the Vampire theorem prover [3] and the Typed Higher-order Form language [4][1]. While it is possible to embed a modal logic in a first order language [5], we have found that creating practical problems in a large ontology often requires quantification over formulas.

SUMO is an axiomatized ontology[2] written in SUO-KIF[3], which has not previously had a formal semantics for its use of syntactically higher-order logic expressions. Our translation to THF provides such a semantics. We do this by using accessibility relations and Kripke semantics [6]. The modal system axioms used are frame axioms [7].

While the bulk of SUMO formulas are first order, there is a significant use of modal logic in formulas that could not be tested until now. SUMO has roughly 20,000 constants and 100,000 formulas. Of those, roughly 8000 are implications, and the style for formalization followed in SUMO is to use implication for conditional expressions, rather than to reduce them to their CNF equivalent.

Of those 8000 implications, 717 of them contains temporal modal operators, which in SUMO involves

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[1]https://tptp.org/UserDocs/TPTPLanguage/TPTPLanguage.shtml

[2]https://www.ontologyportal.org/

[3]https://github.com/ontologyportal/sigmakee/blob/master/suo-kif.pdf

the predicate `holdsDuring` that has a `TimePosition` and a `Formula` as its argument signature, and which states that the given `Formula` holds true during the given `TimePosition`, which is either a point or an interval.

There are 127 formulas that use the `knows`, `believes` or other epistemic operators. There are also 1086 other higher-order formulas that do not use modal operators.

Most relevant for this paper is that there are 114 formulas that use deontic operators, such as that a formula is an `Obligation`, `Prohibition` or that it states a `Permission`.

2. Background and Related Work

Much foundational work in modal logic has been conducted, starting with [8], expanding to different modalities. Work in modal logic has often concentrated on propositional modal logic and often been done independently of automated reasoning, although this too has been changing in recent years [9].

The majority of prior work in this area concerns problems of a few dozen axioms although some are up to 1000 axioms [10]. Additionally, the problems have been developed for the purpose of testing or illustrating issues in modal logic, rather than broader goals of common sense reasoning.

Work in linguistics also provides substantial background for formalization of modal logics in automated reasoning [11].

In prior work we developed an approach for expressing modal logic constructs with SUMO and TPTP [12]. Earlier work [13] has also laid the foundation for expressing SUMO in higher order logic [14, 15, 13]

Prior work [16], concluded that deontic logic can use system D (seriality) if we assume that each obligation does not hold for all worlds (relativistic) or KD45 if it holds for all worlds (absolutist.) Relativism in the context of deontic logic is where multiple obligations do not necessarily hold simultaneously, while absolutism is where the obligations all hold. [17]

For the purposes of this paper, we are testing mainly the absolutist approach for obligation and prohibition/forbiddance, since we want to test the chaining of such axioms to see the resulting behavior. Also, chaining was needed to make obligation examples fire under the system axioms as opposed to modus ponens. We will say more on this later in the method/results section. We want to know how deontic logic can be implemented in a real-world scenario, drawing from the power of a fully axiomatized ontology. We can test how different systems resolve different example problems, including paradoxes such as Gentle Murderer [18], Ross's disjunctive example [19], and Chisholm's contrary-to-duty [17].

3. Outline of the Method

- *reflexive* if $\forall w : wRw$,
- *symmetric* if $\forall w, u : wRu \implies uRw$
- *transitive* if $\forall w, u, q : wRu \wedge uRq \implies wRq$
- *serial* if, $\forall w : \exists u : wRu$.
- *Euclidean* if, $\forall u, t, w : wRu \wedge wRt \implies uRt$. Note that by symmetry, it also implies tRu, as well as tRt and uRu)

Figure 1: Modal Frame Axioms

We have a Java-based translator as part of the SigmaKEE system [20] [4] that is the development environment for SUMO, and which translates SUO-KIF into THF. As we learn how to create translations manually from SUMO to THF we modify the automated translator so that we will be able to translate the entirety of SUMO automatically. The bulk of this paper addresses experimentation in manually-created THF problems in quantified multi-modal logic using SUMO concepts.

We assume a standard set of modal logic frame axioms. The modal systems are specified by the relationships on possible worlds (Figure 1). Modal logic systems follow from these frame

[4] see the code in Modals.java at https://github.com/ontologyportal/sigmakee

Listing 1: Seriality of Obligation

```
thf(serial_obligation, axiom,
 ![W1:w]: ?[W2:w]:
   (accreln1 @ s__Obligation @ W1 @ W2)).
```

conditions (Figure 2). We assume the standard modal logic cube and its orderings of modal systems (Figure 3). These axioms are specified in THF as, for example, Listing 1.

- K := no axioms
- D := serial
- D4 := serial and transitive
- D45 := serial, transitive, and Euclidean
- T := reflexive
- B := reflexive and symmetric
- S4 := reflexive and transitive
- S5 := reflexive and Euclidean

Figure 2: Modal Systems

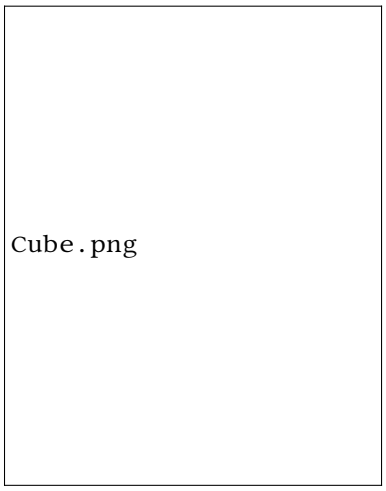

Figure 3: Modal Logic Cube (courtesy Wikipedia)

A first-order formula ϕ can be associated with the modal operators of O, P, F. Each of the operators is defined in terms of the others with the usual axioms that $O\phi \implies P\phi$, $P\phi \implies \neg O\neg\phi$, and $F\phi \implies O\neg\phi$. We also have more specific operators of `Legal`, `Illegal`, and `Promise`, which denote respectively a legal or contractual justification for being permitted, prohibited or obligatory, but these are not used in the context of this paper. In SUMO syntax these are expressed with the relation `modalAttribute`, which relates a `Formula` to a `NormativeAttribute`.

SUMO has a number of modal operators have not typically been the subject of research in modal logic. In addition to specifying that formulas are obligatory, permitted or prohibited, these operators can be parameterized. We have the relation `holdsObligation` that specifies the agent that has the obligation to make a formula true. We also have `confersObligation` that further specifies the authority that confers an obligation upon an agent to make a formula true. Accessibility relations can take 1, 2, or 3 arguments, and are numbered as `accreln1`, `accreln2`, or `accreln3`. The most basic modalities use `accreln1`, since they are agnostic of agents or entities (which would instead use `accreln3` as with the SUMO relation `confersObligation`. Note that `confersNorm` has a `NormativeAttribute` as an argument, so it requires a specialized accessibility relation, `accreln3norm`, different from other modals that have a conferring agent and a confered upon agent.

We use Kripke semantics in translating the syntax of SUMO in TPTP THF. For example “Possession of a driver’s license gives permission to drive.” is expressed in the prefix notation of SUMO (with Davidsonian semantics [21] for the first-order action description, and note that unquantified variables are implicitly universally quantified) as showing Listing 2.

Listing 2: Driver's License

```
(=>
   (and
      (instance ?L DriversLicense)
      (possesses ?A ?L))
   (confersNorm ?L Permission
      (exists (?D ?V)
         (and
            (instance ?D Driving)
            (agent ?D ?A)
            (patient ?D ?V)
            (instance ?V RoadVehicle)))))
```

Listing 3: Driver's license axiom with accessibility relation

```
(=>
  (and
   (instance ?L DriversLicense)
   (possesses ?A ?L ?W1))
  (=>
   (accreln3norm confersNorm ?L Permission ?W1 ?W2)
   (exists (?D ?V)
    (and
     (instance ?D Driving)
     (agent ?D ?A ?W2)
     (patient ?D ?V ?W2)
     (instance ?V RoadVehicle)))))
```

Listing 4: Driver's license axiom in THF

```
thf(ax0,axiom,
   ! [V__A: $i,V__L: $i,V__W1: w,V__W2: w] :
    ( ( s__instance @ V__A @ s__AutonomousAgent )
   => ( ( ( s__instance @ V__L @ s__DriversLicense )
       & ( s__possesses @ V__A @ V__L @ V__W1 ) )
    => ( ( s__accreln3norm @ s__confersNorm @ V__L @ s__Permission @
                            V__W1 @ V__W2 )
      => ? [V__D: $i,V__V: $i] :
          ( ( s__instance @ V__D @ s__Driving )
          & ( s__agent @ V__D @ V__A @ V__W2 )
          & ( s__patient @ V__D @ V__V @ V__W2 )
          & ( s__instance @ V__V @ s__RoadVehicle ) ) ) ) ) ).
```

Translation is performed in two stages, first adding Kripke relations on possible worlds (Listing 3) and then to THF (Listing 4), noting that type guards are added for all variables. We assume rigid domains [22] so statements about class membership or subclasses relationship do not get a world argument.

4. Methodology

We are taking what might be called an empirical approach to deciding which modal systems are appropriate for which deontic operators. We have developed a set of test problems and tried them with several different modal systems to see if they get the anticipated result. The test problems use symbols from SUMO but have been coded by hand.

We primarily used the online version of Vampire 5.0 from SystemOnTPTP[5] and the Interactive Derivation Viewer (IDV) [23] for testing our problems. A good proof used all the problem axioms as well as the expected modal system axioms, and this could be verified by inspecting the proof graph.

We have tested a few examples with each deontic operator and their combinations (see Table 1). These are further divided into several dozen sub-examples. These include *conditionality*, which is whether an obligation requires some other fact. We also tested conjectures that existentially versus universally quantify their arguments. At the top of every manually-created file, there is a file description and summary of results. There is a set of type declarations that contain the deontic operators (*obligation*, *permission*, and *forbiddance*) and accessibility relations.

In each problem file, we state the frame axioms corresponding to K, D, T, B, 4, and 5, which can be commented out or uncommented to create combinations such as D4 or D45. After stating the system axioms, we set up the terms and relations being used in our example. These are also followed by the use of the TPTP language keyword `$distinct` which is a convenience to state that all combinations of its argument are distinct (i.e. non-equal) constants. We declare a world *CW*, or "constant world" [22]. Then, there are axioms that use accessibility relations from one world to another. Finally, the conjecture asks whether there is an accessible world from CW. In queries about Permission we existentially quantify the reachable world, and employ universal quantification over all ideal worlds [24] when querying about Obligation or Forbiddance.

5. Findings

This section reports findings from a series of experiments encoding deontic axioms, covering Obligation (O), Permission (P), and Forbiddance/Prohibition (F), as THF problems derived from SUMO and evaluated using Vampire. The experiments proceeded through a progression of encoding choices: whether norms are followed or not (which hereafter we will call "Norms followed" and "Contrary to Duty" or "CTD"), the form of the conjecture (existential vs. universal quantification over worlds), and whether the modal statement is conditional on some other formula being true (*conditionality*.) Each design decision surfaced interactions with the modal systems under different systems (K through S5), and the accumulated findings converge on a minimal adequate system for each deontic operator.

The main findings of interest were in regard to testing the Obligation deontic modality (Table 1). the *seriality* axiom (which Systems D and D4 use) required an existential conjecture to reach refutation, and were not strong enough to provide a proof for a universal conjecture. While universal conjectures are needed to fully capture the nature of obligation, an existential is entailed by a universal and is also correct, although weaker. The target desired system is D4 for obligation.

Another issue with *seriality* is that it guarantees at least one accessible world, meaning it can provide the relation needed to reach an ideal world under obligation if that obligation statement does not require *conditionality*.

Forbiddance/Prohibition was based on Obligation since they are both universal in accessible worlds, so they can be chained knowing that all worlds must hold for a given condition before going on to the next obligation (*conditionality*.)

On the other hand, Permission was more straightforward since it does not need to support chaining of permissions. It is also existential in accessible worlds, so if it can only access some worlds, this means that there exist worlds where it did not, so it does not imply an accessibility to other worlds.

[5] https://tptp.org/cgi-bin/SystemOnTPTP

Combination	Norms followed	Contrary to Duty
Unconditional + universal	K	Wrong refutation under D45
Unconditional + existential	D (too low)	Wrong refutation under D4
Conditional + existential	D4	Timeout (correct)
Conditional + universal	D45 (want D4)	Timeout (correct)

Table 1
Summary of Obligation behavior across conjecture and conditionality variants

5.1. Universal vs. Existential Conjectures

A question in the encoding of Obligation concerns whether the conjecture should be stated universally or existentially over ideal worlds. Conceptually, obligation is a universal modality: in standard possible-world semantics, a proposition is obligatory if and only if it holds at all deontically ideal worlds accessible from the current world. This mirrors the relationship between necessity and universal quantification over accessible worlds. However, System D is existential by nature since it guarantees the existence of a serial successor but makes no claim about all successors. A universal conjecture therefore creates a conceptual mismatch with the D-family of systems: the systems are asked to prove a universal claim using only existential closure properties. In practice, this forces the prover to a stronger system (D45) to achieve refutation with only the Euclidean axiom under a universal conjecture with *conditionality* of obligations.

This tension was resolved pragmatically. Since any universal claim entails its existential counterpart, the existential conjecture functions as a computationally tractable necessary consequence of the intended universal semantic. The System D4 result is then understood as follows: the system is not proving that the obligation holds at all ideal worlds, but rather that it holds at some world reachable by the composed accessibility relation, a sound and useful weaker result. The full universal character of Obligation and Forbiddance is deferred to the contrary-to-duty (CTD) [25] sanction structure. For Permission, the conjecture is expected to be existential, which matched seriality under Permission.

5.2. Obligation: Unconditional vs. Conditional Deontics

Example	Norms followed	Contrary to Duty	Notes
1a_Obligation_univConj	D45	none	Unable to make D4 fire
1b_Obligation_polymorphic	N/A	N/A	Error: Does not compile in ATP
1c_Obligation_parallel	K	D45	Either bypasses systems, or Euclidean incorrectly refutes
1d_Obligation_nHopsConj	K	none	Partial success: Conjecture only works if N obligations are valid
1e_Obligation_existNHops	K	none	Bypasses systems
1f_Obligation_existConj (main)	D4	none	SUCCESS: Works with transitive without reflexive
1g_Obligation_UO1_existConj	D	D4	Issue: Works for D, but wrong refutation by D4 for CTD
1h_Obligation_UO1_univConj	K	D45	Either bypasses systems, or incorrectly refutes with Euclidean axioms
1i_Obligation_singleO	K	none	Bypasses all system axioms

Table 2
Full Obligation results: As with previous table, we want **Norms followed** to meet a refutation at either D or D4, and we want *Contrary to Duty* to never reach refutation

Initial experiments with *unconditionality* of Obligation axioms, of the form "it is obligatory that agent A perform action x", revealed a fundamental sensitivity to the choice of conjecture. *Ideal worlds* are situations where a modal operator (such as O, P, or F) hold true. When the conjecture was stated

universally (i.e., x holds at all accessible ideal worlds), System K was sufficient to produce refutation by simple Modus Ponens reasoning, bypassing the modal system axioms entirely. This outcome is incorrect for the purposes of evaluating modal systems, since the test provides no discriminating information about which systems are adequate.

Reformulating the conjecture to be existentially quantified (i.e., there exists some accessible world at which x holds) to use System D (seriality) ensures the existence of at least one accessible ideal world. However, D is too permissive for chained obligations (i.e. $O\phi_1 \implies O\phi_2 ...$), and since it only guarantees a single ideal successor, a sequence of two conditional obligations cannot be verified without transitivity. The decisive finding emerged when both *conditionality* and existential quantification were combined. Conditional Obligation axioms, of the form "if x then it is obligatory that y", cannot apply unless their antecedent holds in the current world. This prevents the prover from bypassing the accessibility structure. Under this encoding, *Norms Followed* (positive case) produced refutations at System D4 and above, while *Contrary to Duty* (negative case) timed out as expected. Transitivity, contributed by the modal system "4", is essential, since seriality alone reaches one ideal successor and transitivity allows the chain of conditional obligations to be composed across worlds, reaching the terminal world where the conjecture can be verified.

Although D45 also achieves refutation for *Norms Followed*, it is stronger than necessary and may lead to issues when sanctions are used, since an ideal world with met obligations should not be accessible with a parallel sanction world where obligations were not met.

In contrast, unconditional obligation leads to refutation with System D4 under *Contrary to Duty*. In this case, seriality is actually too strong: seriality guarantees at least one accessible world, allowing CW to reach the first obligation, which then holds unconditionally and proceeds to the next obligation. This is problematic, and is still being investigated.

5.3. Permission: System K Is Sufficient

Example	Norm Followed	CTD	Notes
2a_Permission_chained_univConj	D45	none	Same issue as 1a
2b_Permission_chained_existConj	D4	none	Incorrectly behaves like O
2c_Permission_nonChained_existConj	D	none	Works with Sys. D
2d_Permission_existConj_weakP (main)	K	none	SUCCESS: Works with Sys. K
2e_Permission_existConj_strongP	D	D	Refutes without fact axiom

Permission was encoded as a possibility modality, since an agent is permitted to perform X if there exists at least one accessible ideal world at which X holds. This is semantically an existential claim, and the experimental results confirm that System D is both necessary and sufficient, providing exactly one guaranteed serial successor. Two conjecture forms were evaluated. A universal conjecture required T as the minimum system. An existential conjecture avoided this allowing System D to be the minimum system to correctly refute *Norms followed*, while System K (lacking seriality) timed out, confirming that the serial axiom is the correct system. The non-chained structure was also tested. With the conjecture stated existentially and axioms in "parallel" form (both obligations anchored to the current world rather than chained), System D refuted successfully and System K timed out. The chained form produced the reverse: System D timed out (correctly unable to reach the terminal world), while System T refuted (incorrectly, via reflexivity). This confirms that the non-chained arrangement of Permissions is the appropriate structure, and that System D is the minimal adequate system.

However, it was later seen that the example Permission axiom itself was universally quantified, just like for *O* and *F*, which is not conceptually correct. A design decision needed to be made on whether to implement Weak Permission or Strong Permission. Weak Permission is when one is permitted to do an action as long as there is no explicit Prohibition or Obligation otherwise, while strong Permission is when a permission is explicitly required separate from the other deontic attributes. [26] For deterministic behavior between the three deontic attributes O, P, and F, weak Permission was chosen. Strong Permission lead to incorrect refutation under CTD.

5.4. Forbiddance: Parallel Behavior to Obligation

Example	Norms followed	CTD	Notes
3a_Forbiddance_univConj	D45	none	Unable to make D4 work
3b_Forbiddance_existConj (main)	D4	none	SUCCESS: D4 works

Forbiddance (Prohibition) was treated as the dual of Obligation and was tested in parallel with Obligation under conditional axioms and existential conjectures. The results closely mirrored those for Obligation: System D4 emerged as the minimal adequate system for chained conditional Forbiddance, with *Norms Followed* refuting at D4 and above, and *Contrary to Duty* timing out.

5.5. Obligation-Permission Bridge

Example	b, D-P, D4-O	b, D4-O	D-P, D4-O	CTD	Notes
4a O-then-P aBridgeTooFar	ref	ref	timeout	timeout	O-then-P, with extra P step: System D is ignored
4b O-then-P stopsAtLastO	ref	ref	timeout	timeout	Behaves same as BTF, but conceptually more correct
4c O-then-P BTF 2HopConj	ref	ref	ref	timeout	Extra P with separate-hop conjecture: Success! Explicit O and P in conjecture and prioritize Sys. D over bridge
4d O-then-P SALO 2HopConj	ref	ref	timeout	timeout	Unlike BTF, requires bridge axiom

A further experiment tested the interaction between Obligation and Permission through a bridge axiom of the form $O(x) \implies P(x)$, which we call the O-P bridge. The first set of examples tested whether two chained obligations could reach a refutation with an additional permission, and were called P1 (or BTF in the examples.) The second set of examples tested without an explicit permission axiom, called P0 (or SALO in the examples.) The initial result for P1 was unexpected: *Norms followed* was refuted under D4 for Obligation, but the Permission system was never invoked. Inspection of the proof revealed that the two Obligation axioms were traversing the world related worlds (CW $\rightarrow$ W1 $\rightarrow$ W2) via the transitive closure contributed by D4, and the bridge axiom was then transforming the final Obligation into Permission by modus ponens, without any Permission-specific accessibility relation being required. Testing two explicit hops was tested, one for permission and one for obligation. This resulted in working with System D under Permission, and when and still used D under Permission when the O-P bridge was reintroduced. This was a success and showed that making the systems work together was possible when the conjecture explicitly separates deontic modalities. For P0, the examples reached refutation with bridge, but timeout if just D4 and D. Unlike the example with an extra Permission, this example requires the bridge axiom (which makes sense since there are only accessibility relations under Obligation without the bridge.) Conceptually, P0 makes sense since it only requires that one have permission to do what is obligated. P1 asks if this goes further and is a stronger claim. If it is decided that P1 is too strong, additional research would be needed to make sure P1 is not refutable and only passes under P0.

Additionally, due to changes in how Permission was found to work under Weak P with System K rather than System D, examples 4E and 4F were tested. Their results are as follows:

Example	Norms Followed	CTD	Notes
4e O-then-P SALO existP	ref	timeout	SUCCESS: Still refutes with fact axiom
4f O-then-P BTF exist P	ref	timeout	The only peculiarity is the lack of transitivity under O.

6. Summary and Adequacy of Minimal Systems

Unconditionality should allow chained obligations, since there are real life examples where an obligation always holds (such as “Thou shall not murder”.) However, no implementation so far has been found that avoids an incorrect refutation under *Contrary to Duty* with *transitivity*, meaning that only System D works safely under *Contrary to Duty*. One possible mitigation would be to avoid posing an unconditional obligation by re-framing the statement like "If X is an autonomous agent, then shall not murder." This makes sense intuitively since the obligation to not murder does not apply to a non-autonomous object, such as a stone for example.

Universality is conceptually correct for Obligation/Forbiddance, but incompatible with System D and required either system K (modus ponens) or System 5 (Euclidean axiom on its own) to work. *Conditionality* worked practically and as expected, with even the strongest systems correctly timing out rather than refuting under *Contrary to Duty*. An existential is entailed by a universal, so while it is conceptually too weak, it is compatible with *seriality* (Systems D and D4), so it has pragmatic value.

If, in practice, existential conjectures are needed for all three deontic modalities, then O, P, and F will need to be differentiated by sanctions. These findings establish a stable encoding baseline for the subsequent contrary-to-duty (CTD) experiments, in which sanctions are modeled as secondary obligations triggered by the violation of primary ones.

All problems and proofs are available at https://github.com/ontologyportal/sumo/tree/master/tests/TPTP/feener_CHAD_examples

7. Future Work

The next phase of this work consists of replicating each of the tests we have coded manually in THF and instead specifying them in SUO-KIF and having our automated translator convert them to THF and validate that we get the same proofs. This will also present a challenge to Vampire as instead of a few dozen axioms we can scale up to including the full set of 100,000 in SUMO. However, there is hope that this should scale well due to Vampire including the SInE algorithm [27].

Declaration on Generative AI

The author(s) have not employed any Generative AI tools.

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